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Nonlinear Control Methods for High-Energy Limit-Cycle Oscillations

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Limit-cycle oscillations, as they appear in high-performance fighter aircraft, remain an area of scrutiny by the aerospace community. Tools for the simulation and prediction of the onset for limit-cycle oscillations have matured significantly. Surprisingly, less progress has been made in active control methodologies for these inherently nonlinear dynamic phenomena. Even in the cases where it is known that limit-cycle oscillations may be observed in particular flight regimes, and active control methodologies are employed to attenuate response, there are very few analytical results that characterize the stability of the closed-loop system. In part, this may be attributed to the difficulty in understanding the nature of the contributing nonlinear structural and nonlinear aerodynamic interactions that account for the motion. Recent progress made by the authors in the derivation, development, and implementation of nonlinear control methodologies appropriate for a class of flutter problems is reviewed. Both analytical and experimental results are summarized. Directions for future study and, in particular, remaining technical barriers that must be overcome, are summarized.

I. Introduction

FOR the past decade, a concerted effort has been made to understand the fundamental mechanics represented by low-dimensional models for complex aeroelastic structural response, including bifurcation phenomena, limit-cycle oscillations, and chaos. Representative research in this field can be found in Refs. 1 and 2. During the same time period, parallel studies have led to a theoretical foundation for a number of nonlinear control methodologies that appear, in principle, applicable to this class of nonlinear systems. Representative examples of geometrically nonlinear methods can be found in Refs. 3 and 4. One can appreciate the significance of this body of research simply by noting that nearly every modern textbook on nonlinear control now contains preparatory material for geometric methods.[5–7] It is usually implicitly understood that other noncanceling control methodologies, such as backstepping, for example, are applied to systems that have been nonlinearly feedback linearized, insofar as it is possible.[8]

Still, it is remarkable that there has been little communication between the two intrinsically related fields of 1) low-dimensional, nonlinear aeroelastic models and 2) nonlinear geometric control methods. The line of research followed in Refs. 1 and 2, for example, leads the field in understanding the fundamental mechanisms by which pathological aeroelastic response may manifest itself, but does not exploit nonlinear control methodologies for this inherently nonlinear phenomena. Some linear control methods have been employed for this class of problems. The approach taken in Ref. 9 is typical of the linear control approaches for small-amplitude nonlinearities. Similarly, some conventional least-mean squares and least-squares algorithms have been applied.[10] These approaches, however, do not account for the underlying geometric structure inherent in the nonlinear governing equations. At the same time, some nonlinear control methodologies, such as those described in Refs. 3 and 4, have been criticized by some engineers as being sterile and only applicable to low-dimensional, academic problems.

In 1995, in collaboration with NASA Dryden Flight Research Center, these authors[11–16] set about to investigate to what degree genuinely nonlinear control methodologies would be amenable to this nonlinear class of aeroelastic response. In their previous publications on this topic, the authors have 1) derived low-dimensional nonlinear models that are accurate for the class of large-amplitude, limit-cycle oscillations (LCOs) discussed herein, 2) derived Lie algebraic full and partial feedback linearizing control, 3) derived nonlinear model reference adaptive control (MRAC) methodologies to control LCOs, and 4) experimentally tested the derived control laws to validate them for this class of nonlinear aeroelastic phenomena. In this paper, we summarize the relative merits of these methodologies, we review their evolution historically in the context of recent trends in research in nonlinear control theory, and we discuss future research issues and directions. Insofar as this paper reviews the overall context of the development and performance of these methods, we refer to appropriate references for the theoretical details.

II. Technical Background

Flight tests of several advanced fighter aircraft have revealed LCOs for certain attached wing store configurations (see Denegri and Cutchins,[17] Chen et al.,[18] and Stearman et al.[19]). The detailed mechanism that leads to these LCOs is not well understood, but it is virtually certain that a full exploration will involve coupled aerodynamic and structural nonlinearities. The reliable, accurate, and cost-effective prediction of nonlinear fluid–structure-interaction (FSI) response remains one of the most challenging problems to analysts and experimentalists in the aeronautics research community. Among these response regimes one may include classical divergent flutter and various manifestations of LCOs. Analysts sometimes simply define LCO as an isolated periodic response of the underlying dynamic system. On the other hand, flight test engineers often choose to distinguish among classes of LCO as typical or nontypical, depending on whether the sustained amplitude of entrained motion

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does, or does not, progressively increase with increasing Mach number. However, although the nomenclature used to distinguish these phenomena varies among research communities, and even among researchers within one community, the importance of characterizing these phenomena cannot be overstated. LCO have been observed on most fighter aircraft. They have been quite frequently associated with classes of store configurations and wing tip missile launchers on the F-16 and F/A-18. Wing LCO has also been observed in the B-1a wing.[20]

For some time, there has been a mild debate among aeroelasticians regarding the mechanisms by which LCO originate in these high-performance aircraft. There seems to be a consensus among researchers that LCO is an inherently nonlinear response regime. However, several researchers disagree as to the predominant cause of certain observed classes of LCO. In F-16 flight tests, for example, LCO response is most frequently associated with high-subsonic or low-supersonic Mach numbers. Some researchers attribute the observed LCO response to nonlinear aerodynamic forces associated with shock-induced trailing-edge separation in the transonic regime. In some of these cases, accurate predictions of LCO response can be achieved using nonlinear aerodynamic models coupled with linearly elastic structural models.[21] At the same time, Denegri[22] has observed the following through flight-test experiences of the F-16:

> Theoretical flutter analyses . . . adequately identify flutter or LCO sensitive store configurations and their oscillation frequencies. In addition, a strong correlation between flight test response and the modal composition of the analytical flutter mechanism is evident. However, the linear analysis fails to provide insight into the oscillation amplitude or onset velocity which are of primary importance for external store certification on fighter aircraft.

In addition, numerous researchers insist that the response cannot be adequately modeled with only nonlinear aerodynamics. There exists plentiful anecdotal flight-test evidence that LCO is observed at flight speeds well below the transonic regime. Motivated by these observations, some researchers have focused on studying the role of nonlinear structural response in LCO.[11,12,18] Recently, laboratory load testing of full-scale F/A-18 wings (R. Yurkovich, informal correspondence based on McDonnell Douglas Corporation 97A0106 "F/A-18 C/D Tip Missile Aerodynamic Effects on Flutter Speed," May 1998) has shown that structural nonlinearities are evident and appear in both load stiffening of the wing and in load-deflection hysteresis effects. Both of these classes of nonlinearities have been associated with LCO in prototypical mathematical models.

III. Control of LCO for Low-Speed Flows

In view of the preceding discussion of the diversity of potential causes of LCO and the poorly understood nature of some classes of LCO, it is clear that any attempt to control actively LCO faces substantial challenges due to modeling uncertainty. We emphasize that when we speak of developing a control methodology in this paper, we do not simply mean designing an ad hoc controller and empirically evaluating its performance. Rather, we restrict our attention to cases in which some type of analytical stability guarantee can be made for the nominal governing equations. Admittedly, in some cases, the class of nonlinearity considered is limited to make the problem tractable. Nevertheless, these considerations help define the strength and limitations of current control methodologies and identify control theoretic questions that remain open.

As discussed more fully herein, the authors have had considerable success in the derivation of geometric, nonlinear control laws for the control of high-energy LCOs. In our previous work, the forcing and aerodynamic information has been largely obtained based on global measurements or characterizations of the overall lift and pressure. Throughout this paper, we focus on a class of LCO that is associated with geometrically nonlinear structural response in low-speed flows. It is demonstrated that even with limited knowledge of aerodynamics, useful nonlinear controllers can be derived that outperform conventional linear controllers.

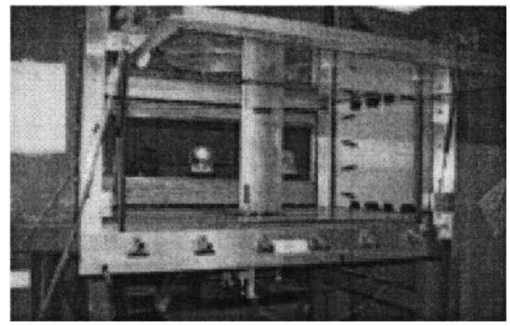

Fig. 1 Aeroelastic control testbed, 3 × 4 ft low-speed wind tunnel at TAMU.

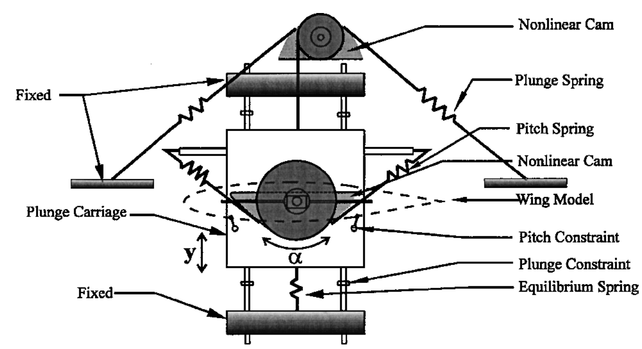

Fig. 2 Schematic of nonlinear cam support, experimental testbed TAMU.

Figure 1 shows the experimental facility developed at Texas A&M University (TAMU) that has been used to test various control methodologies studied by the authors. A model support system (see Fig. 2) has been developed to provide experimental investigations of nonlinear aeroelastic response, as well as control of such response.[13,14] The support system permits pitch and plunge motion for a mounted wing section. For studies of control of nonlinear aeroelastic response, the wing includes a full-span trailing-edge flap. The plunge degree of freedom is provided by a carriage that translates. Pitch motion, independent of plunge motion, is provided by rotational cams that are mounted on this carriage. The pitch and plunge responses are prescribed by the properties of the springs and cam shapes used in the experiment.

The shape of each cam, stiffness of the springs, and pretension in the springs can be tuned to dictate the nature of the nonlinearity. With this approach, these cams provide tailored nonlinear stiffnesses in the pitch and plunge degrees of freedom. Other physical properties such as the eccentricity of the aerodynamic center, the mass of various system components, the mass eccentricity, the moment of inertia of the wing, the stiffness characteristics, and the wing shape can be easily modified. The resulting system is characterized by a multiparameter-dependent system of coupled nonlinear ordinary differential equations. System response is measured with accelerometers and optical encoders mounted to track motion in each degree of freedom. As shown in Figs. 3 and 4, large-amplitude LCOs are obtained over a wide range of parametric variations of elastic axis location and flow speed. Excellent correlation between the open-loop LCO predicted by the model and the measured experimental results is achieved.

The resulting system is exceptionally well suited for the study of nonlinear control methodologies for many relevant flutter problems, including store-induced flutter. The models derived in Refs. 11 and 12 are low-dimensional, accurate, and capture the essential physics of the flutter response for the experimental regime under investigation. Moreover, the nonlinearities induced by the nonlinear test apparatus are smooth and amenable to analysis by Lie algebraic and certain nonlinear MRAC techniques. Finally, the authors have been able to derive linear controllers that stabilize low-energy, small oscillatory motions of this nonlinear system. However, if these linear controllers are employed when the system response is entrained in

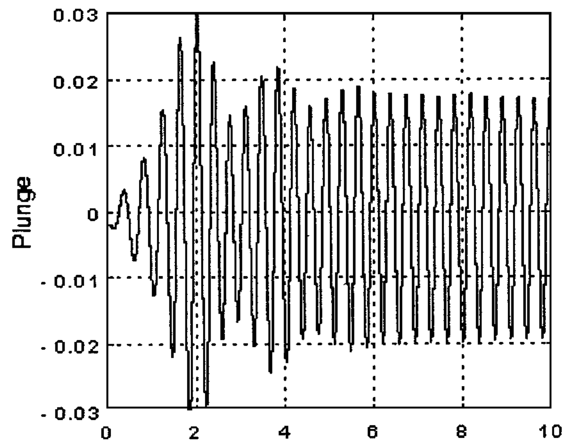

Fig. 3 Measurements of large-amplitude limit-cycle response (flow speed = 14 m/s).

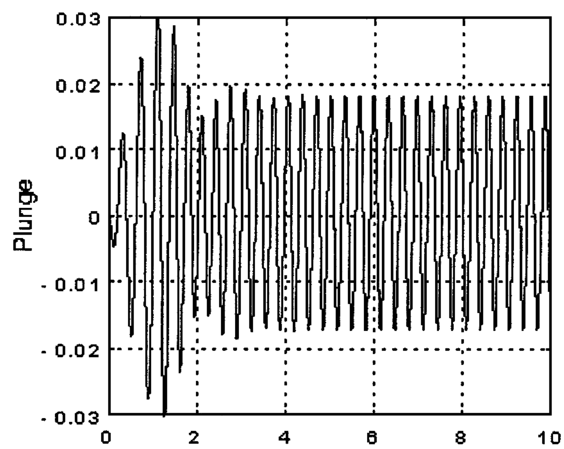

Fig. 4 Predictions of large-amplitude limit-cycle response (flow speed = 14 m/s).

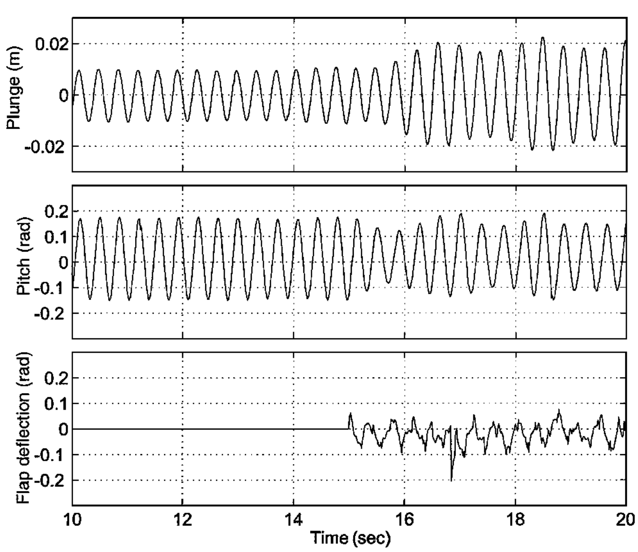

Fig. 5 Closed-loop response to conventional linear quadratic Gaussian controller.

the high-energy LCO, the results are not predictable and generally unstable, as shown in Fig. 5.

A. Lie Algebraic Control Methodologies

Over the last two decades, one of the most elegant theories to emerge for the control of nonlinear systems has been the class of geometric and Lie algebraic control methods. A full discussion of the fundamental theory for this approach can be found by Isidori.[3] Perhaps the most attractive feature of the Lie algebraic methods is the natural and rigorous manner in which certain classical concepts from linear system theory can be lifted to a nonlinear framework. That is, such common notions from linear system theory as controllability, observability, and stabilizability all have natural generalizations in this framework, as do controllability and observability gramians, for example. Moreover, there also exist (at least in principle) simple methods to derive static controllers for some systems that transform the nonlinear system into one that is linear and easily controlled.

On the other hand, some of these features that are attractive in principle are often quite difficult to realize in practice. If an accurate, detailed plant model is available, and if it is (partially) feedback linearizable, the explicit derivation of the controller can be hopelessly complex for all but the lowest dimensional problems. Even more important, the success of the feedback linearization hinges in a crucial way on the exact cancellation of certain nonlinearities appearing in the open-loop model. If the nonlinearities are imprecisely known, the performance of the classical Lie algebraic control method can suffer, which leads in severe cases to instability.

As we will show, these generic advantages and disadvantages of the Lie algebraic control methods are evident in their application to our LCO control problem. Historically, they were the first nonlinear controllers developed to address the LCO response in the experimental facilities. The control methodologies in Refs. 11 and 12 provide detailed derivations of full and partial feedback linearizations for this class of aeroelastic system. In addition, Ref. 15 derives an adaptive feedback linearization method that seeks to address some of the limitations encountered in approaches studied in Refs. 11 and 12.

We first consider typical response for the feedback controllers based on partial feedback linearization without adaptation. As discussed in detail in Refs. 11 and 12, the asymptotic stability of these controllers can be proven, at least locally. These stability theorems say, roughly speaking, that for initial conditions close enough to the trim condition, the response should return to the original trim conditions. Figure 6 shows the results of a typical test for this class of controller.

In Fig. 6, the open-loop flutter velocity is 13 m/s. That is, the onset of flutter occurs at a wind-tunnel velocity of 13 m/s. As shown in Fig. 6, the pitch amplitude maintains a peak-to-peak response over 25 deg in the open loop, while the wind tunnel speed is increased to 17 m/s. With the application of the partial feedback linearizing control, flutter is attenuated rapidly in all cases. However, the

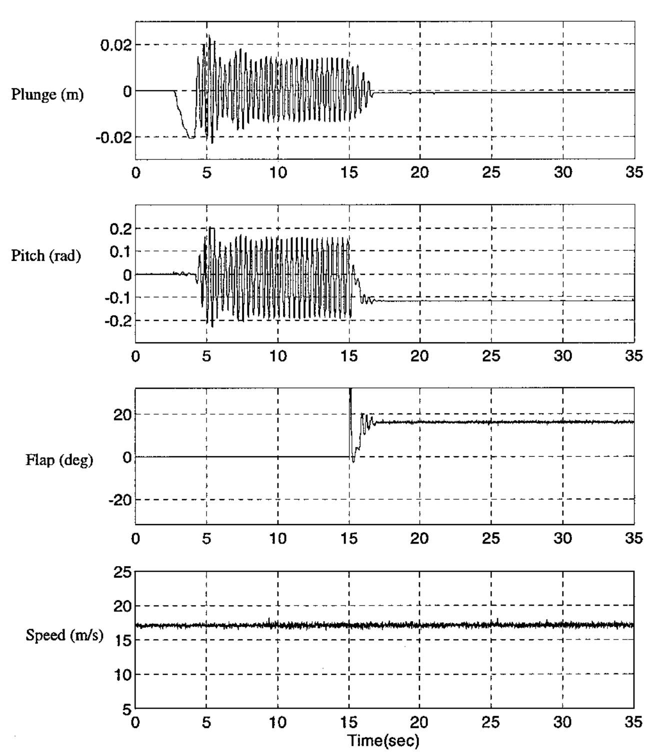

Fig. 6 Closed loop response to Lie algebraic nonlinear reconfiguration.

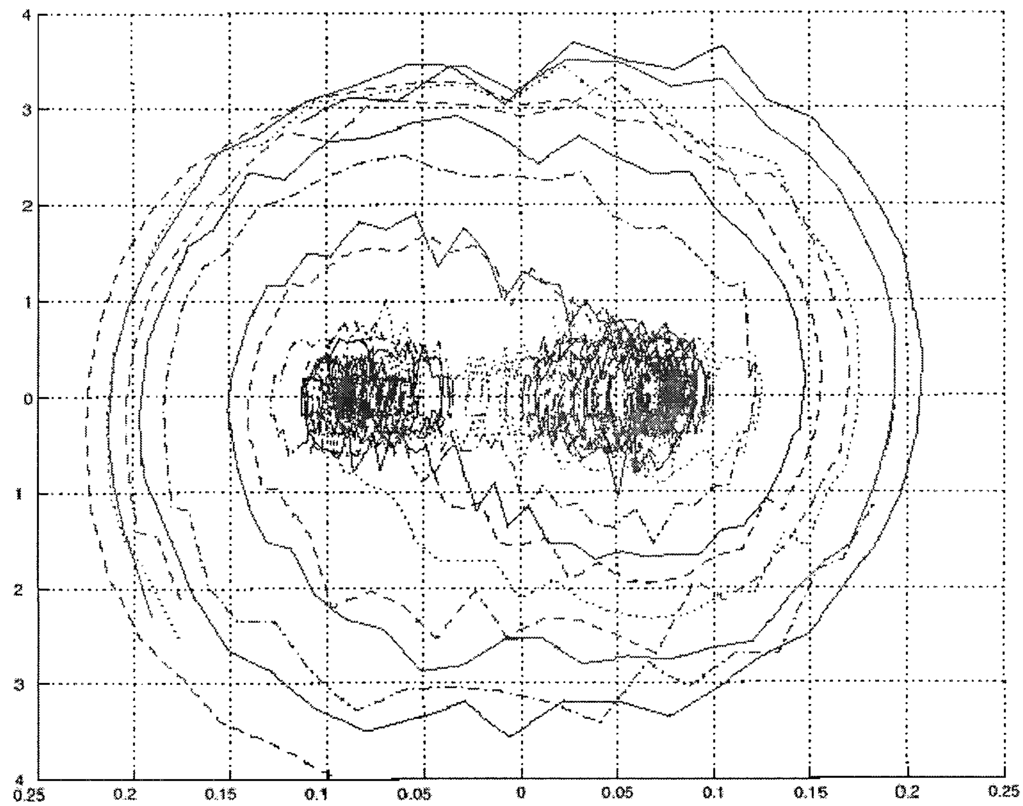

Fig. 7 Phase portrait of multiple equilibria, 15 m/s; intertwining of trajectories to stable equilibria.

wing–elevon system seeks one of two stable equilibrium conditions and terminates the limit cycle. Note that this result is not consistent, strictly speaking, with the theoretical analysis in Refs. 11 and 12 that guarantees local asymptotic stability about the zero (trim) position. However, the deviation between theory and experiment is rather simple to analyze and may be traced to the generic features of the Lie algebraic control methods discussed. These methods require exact cancellation of certain nonlinearities in the theoretical analysis. Because the model is only a low-dimensional polynomial approximation of the actual nonlinearities measured in the system, exact cancellation does not occur. It is possible to study the location of experimentally determined equilibria that result from a mismatch in cancelling feedback derived from a simple polynomial model and the actual experiment. Experimentally observed equilibria will occur at zeros of the associated residual dynamics. Figure 7 shows a phase portrait of the trajectories measured experimentally and shows good agreement with the location of the induced equilibria associated with inexact cancellation. The imposition of the partial feedback linearizing controller actually introduces two adjacent, stable equilibrium configurations. It is interesting and useful that these equilibria are stable and persistent under a wide range of operating conditions.

Whereas the stabilization of limit cycles up to 22 m/s (see Fig. 8) when the onset of LCO occurs at 13 m/s is desirable, it would be further desirable to stabilize about the zero-pitch, zero-plunge open-loop (trim) condition. We now consider adaptive Lie algebraic control methods developed in Ref. 15 that achieve this goal. For example, Fig. 9 shows the large-amplitude LCO that ensues when the flow speed is fixed at 16 m/s. The onset flutter speed for this experimental configuration is again 13 m/s. As shown in Fig. 5, the pitch amplitude maintains a LCO response that exceeds 25 deg peak-to-peak in the open loop. With the application of the adaptive Lie algebraic feedback linearizing control derived in Ref. 15, the LCO is attenuated completely in 2.5 s (Ref. 16). This method is a variant of the adaptive control methodology for feedback linearizable systems first proposed by Sastry and Isidori.[23]

In conclusion, we can make the following observations regarding the nonlinear Lie algebraic control methods, for both the nonadaptive and adaptive cases.

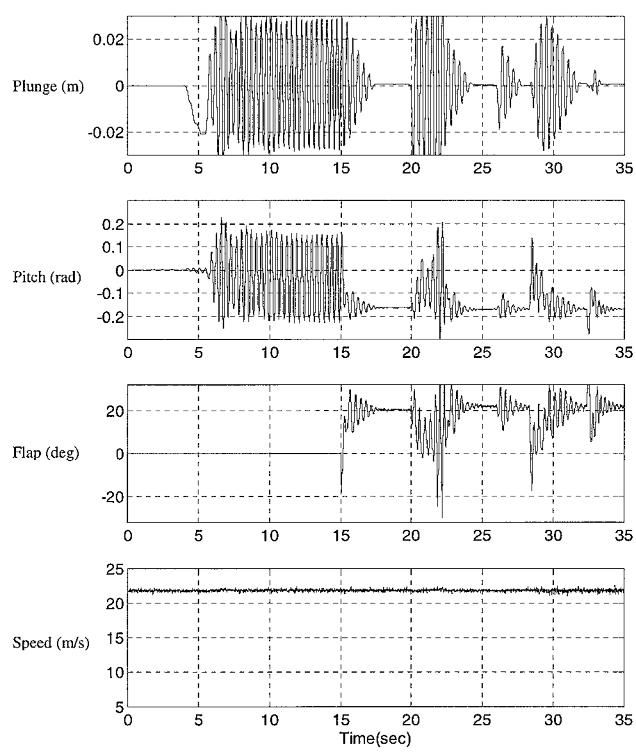

Fig. 8 Time histories for partial feedback linearization, 22 m/s.

1) The analytical stability results about zero-pitch, zero-plunge (trim) conditions for nonadaptive partial feedback linearizing controls derived in Refs. 11 and 12 are not always realizable in practice due to modeling errors.

2) The selected nonadaptive partial feedback linearizing controller induces adjacent equilibrium conditions that are robust; they are attractive and stabilizing over the entire tested range of flow velocities from 15 to 22 m/s.

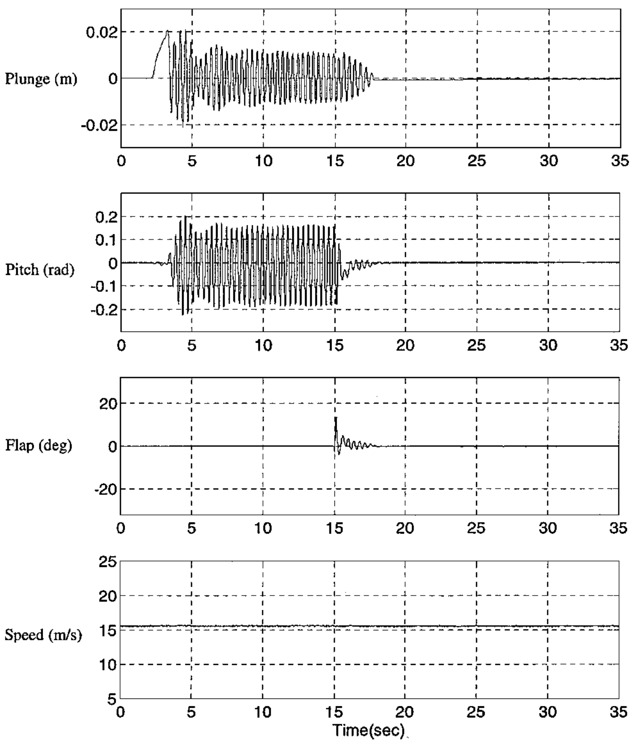

Fig. 9 Closed-loop response to adaptive feedback linearization.

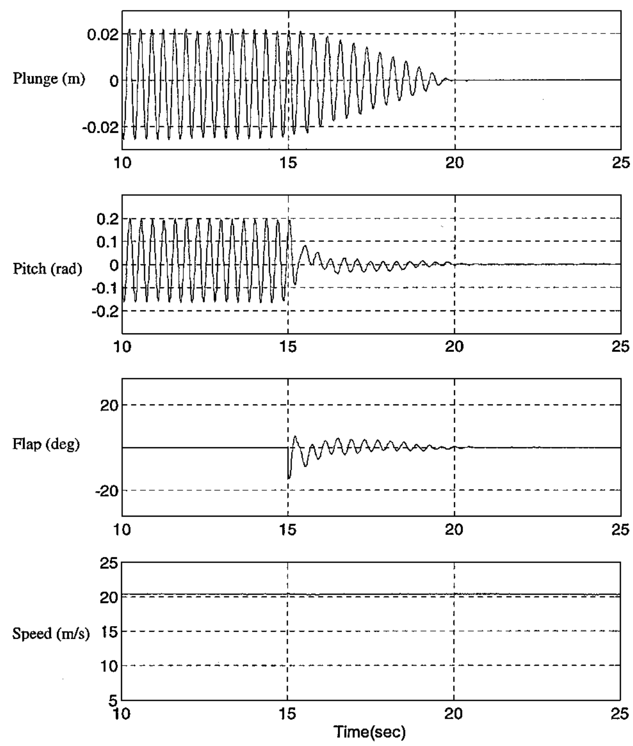

Fig. 10 Closed-loop response to nonlinear MRAC controller.

3) The reconfiguration control in part 2 performs well, but it is not currently understood how to make this a rigorous design methodology. Current research into this matter is ongoing.

4) The adaptive variants of the Lie algebraic control methodologies will stabilize the response about the zero-pitch, zero-plunge trim condition up to 16 m/s. Above that flow velocity, however, the experimental results are unpredictable.

B. Nonlinear MRAC Methodologies

Because the classical (partial) feedback linearization techniques depend on explicit knowledge of the structure of certain nonlinearities in the open-loop equations of motion, there has been a significant thrust within the control community to derive methodologies that are more robust with respect to uncertainty in the structure of nonlinearities. This shift in emphasis has occurred over the past decade. Several alternative control approaches have been studied including nonlinear H^∞ methods, backstepping and its variants, and several classes of nonlinear model reference adaptive control. We briefly discuss some recent results of nonlinear model reference adaptive control to the aeroelastic testbed discussed earlier. Figure 10 shows the performance of one such nonlinear MRAC controller based on the work in Ref. 24 and the references therein.

Generally, we have found that the nonlinear MRAC controller requires somewhat less control actuation, and the settling time for the response is faster. If this was the only difference between the two controllers, one might be tempted to conclude that there was only a modest contrast between the two strategies. However, our study indicates that for higher flow velocities that the nonlinear MRAC appears to be significantly more robust than the adaptive Lie algebraic controller. For example, Fig. 10 shows the response of the nonlinear MRAC controller for a flow velocity of 20 m/s. We hesitate to draw strong conclusions regarding the performance of the nonlinear MRAC controller at this point. There are some numerical tuning issues, and for a given choice of learning/adaptation constants, the performance of the controller may be sensitive to the initial conditions used for certain ranges of parameter estimates. Hence, although the results for the nonadaptive partial feedback linearizing controller were generic for all tested initial conditions, the results for the nonlinear MRAC system seemed to have a greater sensitivity to tuning. Further study of these issues is ongoing.

IV. Limitations and Research Needs

Note carefully two features of this collection of closed-loop controllers. First, the energy of the entrained LCO makes control by conventional linear control methodologies problematic. That is, all of the nonlinear control methods considered successfully stabilize the geometrically nonlinear system, at least over a significant range of flow conditions, whereas the linear controller fails to do so (Fig. 5).

Thus, although these analytical and experimental results are quite promising and illustrate the potential of some nonlinear control methods for flutter control, they only begin to address the host of additional questions raised by this research. Specifically, note the following points.

1) The derived control and the experimental verification were restricted to low-speed flow regimes. In this regime, a simple model of the aerodynamic loads was sufficient to achieve closed-loop flutter suppression, despite unmodeled stall and flow separation. It is anticipated that further extension of the flutter margins will require richer, more physically accurate aerodynamic models in the closed-loop control strategy.

2) The derived control and the experimental verification made provisions for nonlinear, structurally induced LCO. Many of the current studies of LCO are based on a response that is dominated by nonlinear aerodynamics.

3) The computational cost and complexity of higher-dimensional aerodynamic models can be prohibitively expensive for real-time feedback control design, synthesis, and application. Explicit consideration of how system model complexity manifests itself in the resulting nonlinear control formulation requires careful study.

Our closed-loop controller results were obtained using a classical quasi-steady-aerodynamic model. However, during the large-amplitude LCO oscillations, peak pitch angles were sufficient to enter stall and flow separation regimes. This known to occur separation is not modeled, but the nonlinear controllers we developed must have been sufficiently robust to stabilize the motion in spite of this model error. Based on experimental evidence, the further extension of the flutter margin requires richer aerodynamic models in our closed-loop models. In short, we seek to emphasize that improved flutter attenuation via active control requires incorporation of aerodynamic models that better model these complex interactions of the fluid field, structure, and control system. These models may be based on the knowledge of the flowfield from detailed computational fluid dynamics (CFD) analysis or full field experimental data. Thus, nonlinear modeling, nonlinear system identification, and nonlinear control studies should be undertaken simultaneously. Furthermore, in the process of implementing control strategies, the effect of structural dynamics on the fluid flow must likewise be considered. In short, we need to develop capabilities to treat not only fluid and structure dynamics, but also their interaction. Hence, the need for systematic methods for the generation and representation of low-dimensional nonlinear models of fluid flow for control methods is clear.

V. Reduced-Order Modeling

It is rather easy to say that novel reduced-order modeling methods are required to further extend the flight envelope of aircraft. The difficulty lies in finding methods that are amenable to closed-loop control derivation. In the following few sections, we summarize some of the host of order reduction methods that have appeared in the literature. These methods typically have arisen in the treatment of structural dynamics, linear control theory, aeroelasticity, and flow control. Following the summary, we suggest two alternatives to reduced-order modeling that seem most promising for the derivation of closed-loop controls.

A. Reduced-Order Aeroelastic Modeling

Whereas it is well-known that enormous strides have been made over the past few decades in CFD, and computational structural mechanics (CSM), somewhat less progress has been made in the development of rigorous methods for the simultaneous simulation of FSI. There are some notable exceptions, of course, where progress has been made in the simulation of FSIs for aircraft. In some cases this progress can be traced to improvements in coupling strategies for existing CFD and CSM algorithms. The progress can likewise be attributed in part to the improvement of moving mesh techniques required to simulate accurately the aeroelastic fluid–structure system.[25–28] Some analysts foresee the potential of advanced modeling methods for predicting these response regimes in problems of aeroelastic and FSI. At the same time, they suggest that mature algorithms that lead to production-ready software will not be generally available, or generally applicable, until as late as 2012.[21]

Nevertheless, even the most robust techniques in development today fall short of our overall goals to provide reliable, accurate, and cost-effective LCO prediction capabilities for high-performance aircraft. It may be argued that years of research and development have been invested in the analytical tools currently used to study FSI in the design of high-performance aircraft. Yet, LCO persists as a poorly understood phenomena. It requires exhaustive flight testing to ascertain safe flight envelopes, despite the best attempts to predict these pathological response regimes. With the most recent contributions to combined FSI simulation methodologies, many researchers feel that the fundamental and inherent technical barrier that must be overcome is one of computational cost. In several recent research forums, including 1) NASA Langley Research Center Nonlinear Aeroservoelasticity Workshop (1998), 2) the Air Force Office of Scientific Research Aerostructural Interaction and Control Workshop (1998), and 3) AIAA 40th Structures, Dynamics, and Materials Conference, Special Sessions on Reduced-Order Modeling for Aeroelasticity (1999), at least one common theme has emerged. There is an immediate and crucial need for reduced-order modeling of aeroservoelastic phenomena, including flutter and LCO, that may be incorporated in the design of high-performance aircraft. Several novel alternatives have been suggested to meet this need in the design and redesign cycle, including, for example, Refs. 29 and 30.

It is by no means obvious, however, that reduced-order modeling techniques appropriate for design will be useful for the realization of closed-loop controls. The reader is referred to the discussion in Refs. 31 or 32 for more details.

B. Reduced-Order Control Methods

Hence, we now consider reduced-order models for flow simulation and control. Despite significant progress in diverse disciplines in control theory over the past decade, the control of fluid flow remains one of the most challenging, unresolved problems in the field. The difficulty that arises in deriving low-dimensional, rigorous, and general methodologies for fluid flows can be attributed to many factors. Perhaps the most significant factors are the inherent nonlinearity of the Navier–Stokes equations that represent the dynamics of the flow and the high dimensionality of typical approximations to these equations. To help calibrate the complexity of the control task at hand, recall that the aeroelastic models employed earlier in this paper were expressed in terms of $\mathcal{O}(10)$ equations. If we acknowledge that reasonably accurate CFD models can require on the $\mathcal{O}(10^6)$–$\mathcal{O}(10^7)$ degrees of freedom, it is clear that significantly different strategies are required for flow control problems.

A wide spectrum of approaches have been investigated to attempt to codify methods for flow control over the years. These methods differ markedly in the rigor of their treatment of the underlying mathematical control problem, as well as the degree to which the techniques have been validated experimentally or computationally. On one hand, there is a collection of work that seeks to solve open mathematical problems of control theory associated with fluid flow. In Ref. 33, for example, proof of the existence of optimal controls is given for systems of partial differential equations obtained via coupling of the Navier–Stokes and heat conduction equations. The careful, and sometimes incremental, nature of this type of research is illustrated by Ref. 34. In Ref. 34, the authors derive a mathematically rigorous proof of the convergence of approximations that are applicable to the systems studied in Ref. 33. Thus, the proof of existence of optimal control predates the proof of convergence of approximations for this class of systems by 20 years. Mathematical control theory is frequently characterized by such careful, steady definition of problem and solution. Over the years, careful and rigorous study of mathematical control theory has likewise focused on other specific problems, including control of driven cavity flow[35] and piezoelectric control of shear layers in Ref. 36. General studies of the optimal control of the Navier–Stokes equations, for various classes of assumptions (boundary control, distributed control, two-dimensional problems, three-dimensional problems, etc.) can be found in Refs. 37–41. Although these studies are to be recommended for their rigor, connectivity to experimental verification is often neglected. At the same time, there has been a concerted effort to improve the degree to which rigorous mathematical control theory, in the spirit of the approaches discussed earlier, can complement and benefit from current research in CFD. Joslin et al.[42] summarize the use of adjoint and optimality systems for some optimal flow control problems. Emphasis is placed on the numerical requirements and demands imposed for the direct simulation of the resulting set of coupled, nonlinear partial differential equations. This approach is discussed in Ref. 43 to study the interaction between control theory and direct numerical simulation for zero-net-mass fluidic actuators.

To appreciate the diversity of the approaches taken by researchers to address flow control, consider the careful experimental work that has appeared in Ref. 44. In Ref. 44, studies of boundary-layer modification and flow control via oscillatory zero-net-mass jets is described. Although the theoretically precise work in, for example, Ref. 41 does not include a single attempt to verify its theory via an experimental protocol, the detailed experimental studies in Refs. 44 or 45 are carried out without recourse to any mathematical control theory.

Thus, the need for the synthesis of rigorous control theory and experimental methods has been noted by several researchers over the years. A key step to this synthesis is the development of lower-dimensional descriptions of flow dynamics for control synthesis (for example, see Ref. 46). Some authors have proposed ad hoc methods based on neural networks[47] or methods based on linearization and order reduction via classical linear system theoretic methods.[48]

Additionally, some reduced-order methods for flow simulation and control have appeared in the literature over the past few decades. Proper orthogonal decomposition (POD) has been utilized in Ref. 49 to derive low-dimensional models for a class of fluid flows. POD is essentially a statistically based order reduction that calculates the singular value decomposition of the covariance operator of output measurements. Recently, in Ref. 50 it was shown that POD methods can be used effectively for generating a reduced-order model for the simulation and control of a class of reactor flows. A similar philosophy, with an experimental orientation, is considered in Ref. 51. In contrast to the POD methods, where the reduced-order model is based on an optimality criteria, other authors have demonstrated that effective reduced-order models can be derived from physical considerations. For example, Ito and Ravindran, in Refs. 52 and 53, have shown via empirical numerical evidence that reduced-order Navier–Stokes simulations can yield accurate reduced-order models. This methodology is similar in philosophy to the assumed modes methods of structural mechanics. In fact, the assumed modes methods for the order reduction of structural systems have been studied in detail (for example, see Refs. 54 or 55) and have motivated analogous strategies for the generation of reduced-order models for aeroelastic studies. Finally, there has been recent anecdotal evidence that wavelets and multiresolution methods can provide an avenue for generating reduced-order models for flow. Elezgaray et al., in Ref. 56, study the dynamical systems obtained via periodic-wavelet approximations of prototypical flow equations. Wickerhauser and Farge[57] likewise study the effectiveness of generating low-dimensional approximations of scalar vorticity fields using wavelets and windowed cosine bases. The authors of this paper have recently studied the use of divergence-free wavelets for the order reduction of experimentally collected velocity fields in Refs. 58 and 59.

VI. Conclusions

There still remain significant technical barriers that must be overcome to extend some of the recent work in Lagrangian reduced-order basis methods to derive closed-loop control for aeroelastic systems. Essentially, the difficulty arises from two causes. First, Lagrangian reduced bases (such as proper orthogonal decomposition bases or solutions of the Navier–Stokes equations) are typically determined from open-loop simulations, or a carefully constructed, small family of related closed-loop simulations. Second, the fluid flow response in the desired closed loop may bear little resemblance to that of a small fixed library of candidate Lagrangian bases. This paper has emphasized the inherent dissimilarity that can be observed in practice between the open-loop flow and closed-loop flow, for example.

Based on these observations, the authors see particular promise in two novel, complementary approaches to achieve rigorous, systematic order reduction and closed-loop control for aeroelastic systems: 1) adaptive reduced basis methods and 2) multiresolution approximation of nonlinear Volterra operators. By adaptive reduced basis methods, we mean the investigation of systematic methods for the generation of, and adaptive selection from, redundant dictionaries of reduced bases. By dictionary we refer to its usage in the context of recent nonlinear approximation theory research. Approximation theorists have recently focused a significant effort in deriving nonlinear approximation methods from highly redundant collections of bases, or dictionaries,[60] and the approach can be likened to this philosophy. We emphasize that the physical realization of this control methodology will culminate in a redundant dictionary of models, as opposed to a dictionary of bases per se. However, determination of the model in real time from the subspace may not be feasible, or necessary. A redundant library of models may be determined and precalculated for the associated reduced-order subspaces. Adaptive subspace selection will be realized in practice as adaptive nonlinear model selection.

Methodology 2 of the preceding paragraph is directed precisely toward achieving efficient reduced-order representations of a class of nonlinear systems that is amenable to adaptive and online control methodologies without explicitly selecting an approximating subspace. Recall that once we have made a particular choice of reduced-order basis, the resulting reduced-order equations obtained via method 1 take the form of a system of coupled, nonlinear ordinary differential equations. Thus, although the particular ordinary differential equations representing the flow depend on the choice of the approximating subspace, the form of the nonlinear governing equations does not change. Moreover, it is well known that there are a number of equivalent representations of the input/output map associated with certain classes of (analytic) ordinary differential equations. If we acknowledge that the underlying dynamics are nonlinear a priori, a reasonable starting point for nonlinear control development is to choose one of the standard nonlinear system parameterizations.

In summary, it is clear that the development of physically accurate low-dimensional, nonlinear models and associated controllers will remain a formidable challenge. While this challenge remains, it is also evident that significant progress has been made.

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